Section 3.5 More Patterns

Curriculum Outcomes	Related Activities	Page in Text
• demonstrate and understanding of the zero product property and its relationship to solving equations by factoring • use concrete materials, pictorial; representations, and algebraic symbolism to perform operations on polynomials • solve quadratic equations by factoring • expand and factor polynomial expressions using perimeter and area models	• explore non-linear curves and summarize patterns for developing and using an equation • graphs are used as a basis for developing the need to solve equations • students investigate how equations can be solved through factoring • factoring using algebra tiles and patterns are developed for some quadratics of the form $ax^2 + bx + c$	130 131 133 134

Dec 1-4:46 PM

Complete all questions from Investigation #7 (Pg.130) in your notebook.

Dec 1-4:46 PM

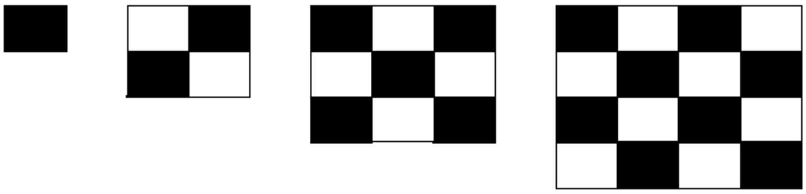
"Part A"

Investigation #7 – More Patterns and Graphs (p 130) **Section 3.5**

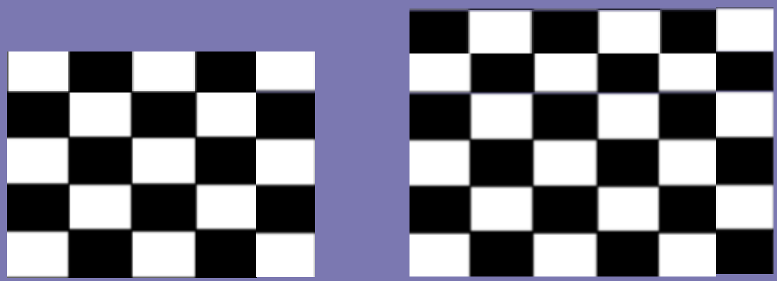
Purpose

To Explore a pattern and use it to create a model for tiling a floor.

Miguel tiled a floor using the pattern shown below. Each square tile is **0.5m wide**



Draw the next two pictures in the pattern



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"Part B"

Questions

1) Complete the table of values below.

Tiles per Side			Area of ONE Tile (m^2)	Total Number of Tiles	Total Area Covered (m^2)
Number	Length of Side(m)	Width of Side (m)			
1	0.5	0.5	0.25	1	0.25
2	1	1	0.25	4	1
3			0.25		2.25
4			0.25		
5			0.25		
6			0.25		

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"Part B"

Questions

1) Complete the table of values below.

Tiles per Side			Area of ONE Tile (m^2)	Total Number of Tiles	Total Area Covered (m^2)
Number	Length of Side(m)	Width of Side (m)			
1	0.5	0.5	0.25	1	0.25
2	1	1	0.25	4	1
3	1.5	1.5	0.25	9	2.25
4	2	2	0.25	16	4.00
5	2.5	2.5	0.25	25	6.25
6	3	3	0.25	36	9.00

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"Part C"

Identify the independent and dependent variables for the above graph.

Independent → ______ Dependent → ______

Construct a graph to show the relationship between the **number of tiles on one side** and the **total area covered**.

# of tiles on one side

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"Part D"

How is the pattern seen in the above graph like the pattern from Investigation 1? How is it different?

Both are discrete data sets. The graph from Investigation 1 forms a linear function (all in a straight line). These do not form a straight line.

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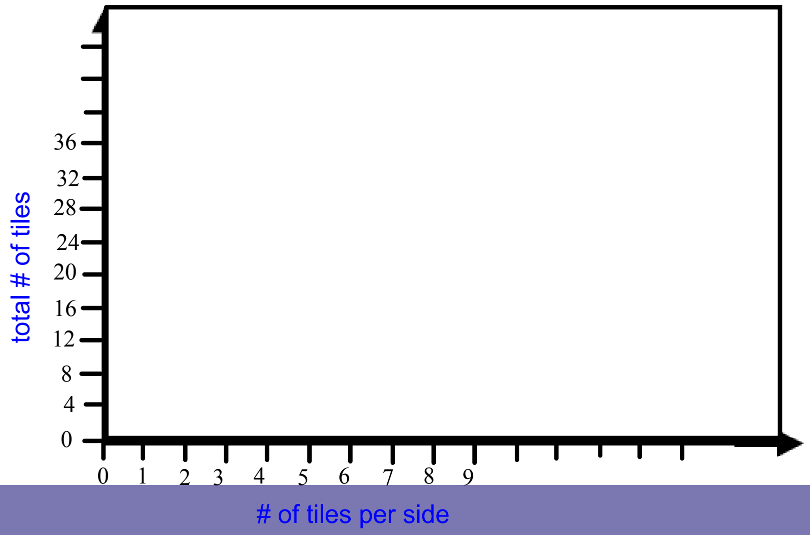
"Part E"

Construct a graph to show the relationship between the **number of tiles per side** and the **total number of tiles**.



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"Part F"

Compare the two graphs from B) and E).
How are the graphs alike?
How are they different?

Both graphs share the same curved shape and both involve discrete data. One graph should appear to be steeper than the other (if the same axis scale is used)

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"Part G"

Describe the pattern that connect the **number of tiles per side** to the **total number of tiles**.
Express your pattern in words.

The total number of tiles is the square of the number of tiles per side. For example, if one side has 3 tiles the total number of tiles is 9 (3^2)

"Part H"

Describe the pattern that connects the total area covered to the total number of tiles.
Express your pattern in words.

The total area is the square of the length of a side. For example, if one side has 3 tiles, a length of 1.5m, the total area is $2.25m^2$ (1.5^2). This would be a linear pattern.

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Answer the Investigations Questions Pg.131 #1 & 2

These predictions can be made with the table of values and/or the graph.

1. (A) 144(0.5)(0.5) = 36m2
   (B) table of values and/or graph
   (C) Estimating directly from the graph doesn't give you confident results
   (D) The accuracy of any method could be checked with any of the other methods. You may have also noticed that a formula or relation could be used.

2. (A) and (B): If 81 tiles are used, then there would have to be 9 tiles per side ($9 \times 9 = 81$). If there are 9 tiles per side, the total side length is 4.5m (9x0.5). The total area is $20.25m^2$ (4.5^2).
   (C) Estimating directly from the graph doesn't give you confident results.
   (D) The accuracy of any method could be checked with any of the other methods. You may have also noticed that a formula or relation could be used.

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FOCUS "K"

Pg. 131

How is the number **of tiles per side** related to the **total number of tiles**?

____

____

Write an equation to connect then **number of tiles per side** and the **total number of tiles**.

____

Write an equation to connect the total area covered and the total number of tiles.

____

a) Use the equation above to determine how many tiles would be needed to cover an area of $150m^2$.

____

____

____

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ANSWERS
FOCUS "K"
Pg. 131

How is the number **of tiles per side** related to the **total number of tiles**?

The total number of tiles = (number of tiles per side) x (number of tiles per side)

Write an equation to connect then **number of tiles per side** and the **total number of tiles**.

$y = x^2$ (y is the total # of tiles; x is the number of tiles per side)

Write an equation to connect the total area covered and the total number of tiles.

Total Area = (area of one tile) x (Total number of tiles)

Area of one tile is 0.5m x 0.5m = $0.25m^2$

Therefore, $y = 0.25x^2$ (y is the total area; x is the number of tiles per side)

a) Use the equation above to determine how many tiles would be needed to cover an area of $150m^2$.

$y = 0.25x^2$ (y is the total area; x is the number of tiles per side)

$150 = 0.25x^2$ Solve for x

$x = 24$

Dec 1-4:46 PM

Use this chart to create graphs for Focus K

Questions

1) Complete the table of values below.

Tiles per Side			Area	Total Number of	Total Area
Number	Length of Side(m)	Width of Side (m)	of ONE Tile (m^2)	Tiles	Covered (m^2)
1	0.5	0.5	0.25	1	0.25
2	1	1	0.25	4	1
3	1.5	1.5	0.25	9	2.25
4	2	2	0.25	16	4.00
5	2.5	2.5	0.25	25	6.25
6	3	3	0.25	36	9.00

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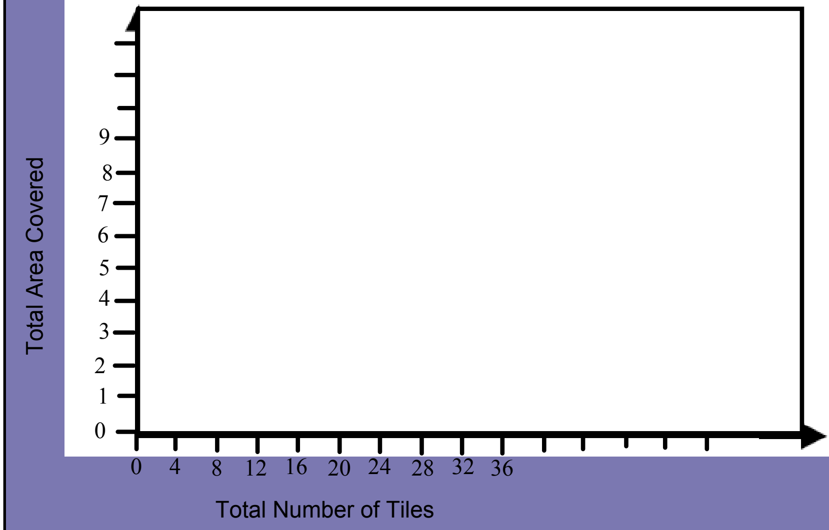
b) Construct a graph that compares the **total number of tiles** used with the **total area covered**.



c) How is the graph above different form the graphs in 2) and 5)? (Parts C and E)

This graph shows the total number of tiles used and not only the number of tiles on one side of the pattern. The equation of the line is y = 0.25x (y = mx + b; the y-intercept is zero)

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Quiz:

For Information: You can use your textbook Pg.130-131

Questions:

1. a. What is the relationship between the number of tiles on one side and the total number of tiles?
   b. Draw a sketch of what the graph of this relationship would look like.
   c. What equation can be used to represent this relationship?
   d. Why should there not be any intercepts on your graph?

2. a. What is the relationship between the number of tiles on one side and the total area?
   b. Draw a sketch of what the graph of this relationship would look like.
   c. What equation can be used to represent this relationship?

3. If there were 14 tiles on one side:
   a. How much area is covered?
   b. What are 2 possible ways to get this answer and which one would you be most confident in and why?

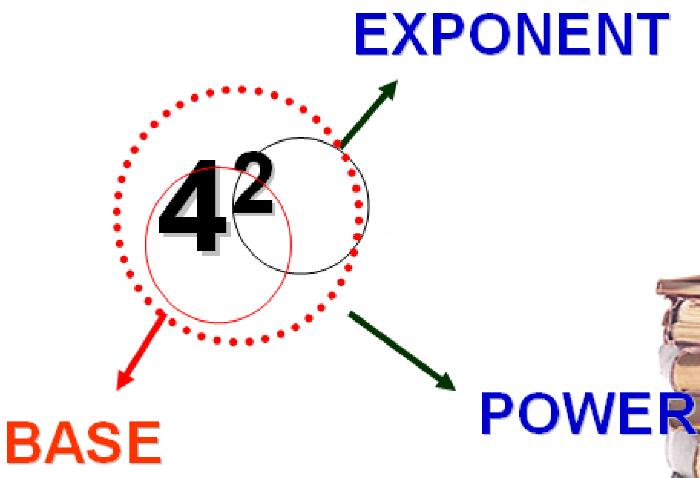
Dec 10-8:29 AM

Review of Exponent Laws

Oct 19-9:13 PM

Oct 19-9:04 PM

POWERS



EXPONENT LAW 1

- PRODUCT OF POWERS

- $n^a \times n^b = n^{a+b}$

- Multiplying powers with the same base

$n^a \times n^b = n^{a+b}$ (Product of Powers)

- To multiply powers with the **same base:**

- KEEP THE BASE
- ADD THE EXPONENTS

- EXAMPLE:
- $3^3 \times 3^4 = 3^7$

Oct 19-9:06 PM

EXAMPE 2:

$$\left(\frac{-4}{5}\right)^2 \times \left(\frac{-4}{5}\right)^3 = \left(\frac{-4}{5}\right)^5$$

Exponent of 1 you don't have to show it

$$(1.1)^3\,(1.1)^2(1.1) = (1.1)^6$$

Oct 19-9:09 PM

EXPONENT LAW 2

- POWER OF A POWER
- $(x^{m})^{n} = x^{mn}$
- Multiply the two powers together.
- Example:
- $(5^{2})^{3} = 5^{6}$

Oct 19-9:10 PM

EXAMPLE – $(x^{m})^{n} = x^{mn}$

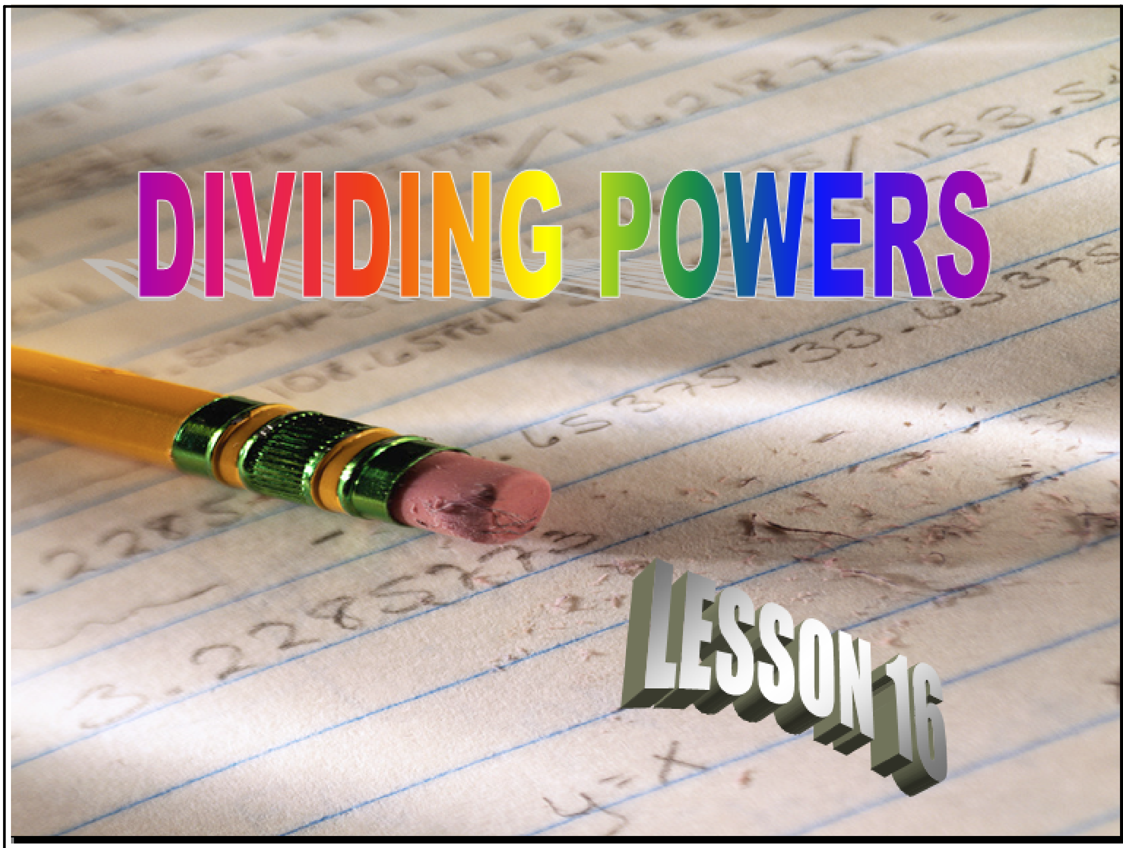
$(3^{2} \times 3^{4})^{3}$

$= (3^{6})^{3}$

$= (3^{6})(3^{6})(3^{6})$

$= (3^{18})$

Oct 19-9:12 PM



Oct 19-9:19 PM

EXPONENT LAW 3

- QUOTIENT OF POWERS

- $x^{a} \div x^{b} = x^{a-b}$

Dividing Powers with the same base

Oct 19-9:20 PM

$x^a \div x^b = x^{a-b}$ (Quotient of Powers)

- To divide powers with the **same base:**

- KEEP THE SAME BASE
- SUBTRACT THE EXPONENTS

- **EXAMPLE:**

$$\frac{3^7}{3^2} = 3^{7-2} = 3^5$$

Oct 19-9:21 PM

TRY THESE

$$4^2 \times 4^3 \div 4^4$$

$$\frac{8^{13}}{8^4}$$

$$\frac{5^4 \times 3^9}{(3^2)^3}$$

Oct 19-9:22 PM

EXPONENT LAWS #1-3

Product of Power $n^a \times n^b = n^{a+b}$

Power of a Power $(x^m)^n = x^{mn}$

Quotient of Power $x^a \div x^b = x^{a-b}$

You need to have these memorized if you already don't!!!!!!

Oct 19-9:22 PM

EXPRESS AS A SINGLE POWER

1) $3^6 \div 3^4$

2) $\dfrac{3^7}{3^2}$

3) $\dfrac{(5^3)(5^4)}{5^2}$

4) $(6^3)(7^8)$

5) $\dfrac{(3^5)(6^2)}{3^2}$

6) $\dfrac{(3^3)(3^4)(5^2)}{(3^2)(5)}$

7) $\dfrac{(4^3)^2\,(4^2)}{(4^2)}$

Oct 20-8:13 PM

ZERO POWERS

Oct 20-8:14 PM

ZERO EXPONENT

X^0 is defined to be equal to 1

$X^0 = 1$, where $x \neq 0$

Exponent Law #4

Any non zero base raised to the exponent zero equals 1

Oct 20-8:16 PM

EXAMPLES

$3^0 = 1$

$(4^2 \times 4^3)^0 = 1$

$$\left(\frac{3^7}{3^2}\right)^0 = 1$$

Oct 20-8:16 PM

EXAMPLES:

ZERO EXPONENTS

$(-5)^0 = 1$

$(2^0)^3 = (1)^0 = 1$

$\left(\frac{2}{5}\right)^0 = 1$

$(-6^{10})^0 = 1$

$-3^0 = -1$

$(-3)^0 = 1$

$-(3)^0 = -(1) = -1$

Oct 20-8:17 PM

NEGATIVE EXPONENTS

x^{-n} is defined to be the reciprocal of x^n

That is $x^{-n} = \dfrac{1}{x^n}$, $(x \neq 0)$

Exponent Law #5

Oct 20-8:16 PM

EXAMPLES

NEGATIVE EXPONENTS

$3^{-1} = \dfrac{1}{3}$

Remember that a negative exponent <u>does not mean</u> a negative number but the *<u>reciprocal number.</u>*

$4^{-3} = \dfrac{1}{4^3}$

Oct 20-8:18 PM

Classwork/Homework

Principles & Process (Orange Text)

Questions Pg.37 #3,4,8,9

Pg.42 #8,11,12

Dec 1-4:46 PM

FOCUS "L"

Pg. 133

Copy & Complete

Julia's math teacher has given her the following table of values. Her task is to find an equation to describe the data.

x	0	1	2	3	4	5
y	3	0	-1	0	3	8

A. Plot Julia's points. Describe the shape of the graph.

B. How is the shape of the graph like the one in Investigation 7? How is it different?

C. Describe the shape of the graph using word symmetry.

D. What is the y-intercept?

E. What are the x-intercepts?

F. Multiply the following: (x-1)(x-3)

Dec 1-4:46 PM

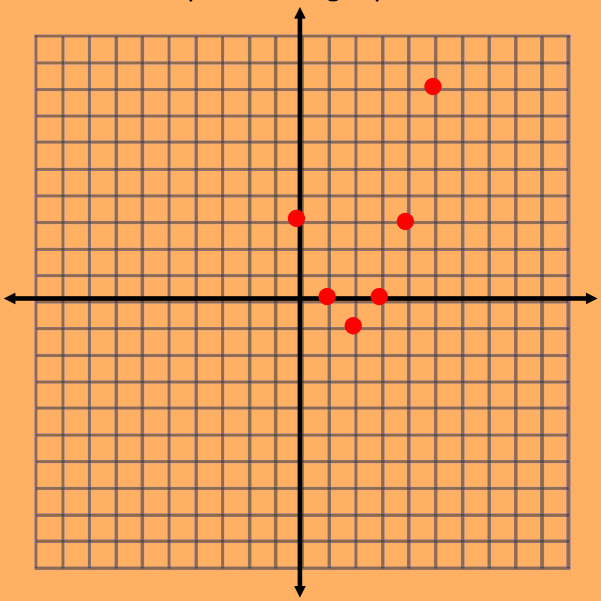
ANSWERS

FOCUS "L"

Pg. 133

A. Plot Julia's points. Describe the shape of the graph.

The graph is "U" shaped



Dec 1-4:46 PM

ANSWERS

FOCUS "L"

Pg. 133

B. How is the shape of the graph like the one in Investigation 7? How is it different?

The graphs in Investigation 7 are similar, but the "U" shape is more complete
with this one. As well, two quadrants are used instead of one.

C. Describe the shape of the graph using word symmetry.

The graph has vertical symmetry.

D. What is the y-intercept?

The y-intercept is 3. At this point x = 0.

E. What are the x-intercepts?

The graph has two x-intercepts; 1 and 3.

F. Multiply the following: $(x-1)(x-3)$

$x^2 - 4x + 3$

The "U" shape graph is a non-linear graph and is called quadratic relation.

Dec 1-4:46 PM

The Distributive Property

Dec 1-4:46 PM

Expanding

- When we multiply a polynomial by a monomial using the Distributive Law, we say we are **expanding the product**.
- **Product** – multiplying to find an answer

Nov 27-10:08 PM

Expanding

- Using the Distributive Law in algebra is called **EXPANDING.**
- **Example: $8x(x - 3)$**

$$8x(x) - 8x(3)$$

$$\boxed{8x^2 - 24x}$$

Nov 27-10:05 PM

Expand

$t(t - x - 2)$

$= t(t) - t(x) - t(2)$

$= \boxed{t^2 - tx - 2t}$

$3(g^2 - 3g + 1)$

$= 3(g^2) - 3(3g) + 3(1)$

$= \boxed{3g^2 - 9g + 3}$

Remember that when you expand, each term is being multiplied by the monomial outside the brackets

Nov 27-10:06 PM

Like Terms

- Remember that in order to add and subtract like terms – you must have the same base to the same power.
- Example: x^2 and x are not like terms because the exponent is different.
- $3x^3$ and $-2x^3$ are like terms because the base and exponent are the same.

Nov 27-10:08 PM

Expanding

$8x(x - 3)$

$8x(x) + 8x(-3)$

$\boxed{8x^2 - 24x}$

$(-5a)(a^2 - 4a - 7)$

$(-5a)(a^2) + (-5a)(-4a) + (-5a)(-7)$

$\boxed{-5a^3 + 20a^2 + 35a}$

Please notice that you can't go any further with these questions because you don't have any like terms.

The bases are the same but the exponent are different.

Nov 27-10:09 PM

Practice:

Principles & Process Text
Pg.67 #13

Dec 7-6:39 PM

Factoring using the GCF

Factoring a product:

You need to split (4x + 8) into groups.
There are 3 different ways that you can equally split up (4x + 8). They are:

- Four groups of (x + 2) (which is expressed as 4(x + 2)
- Two groups of (2x + 4) (which is expressed as 2(2x + 4)
- One group of (4x + 8) (which is expressed as 1(4x + 8)

All of the these are considered FACTORS of (4x + 8)

$$(4x + 8) = (4)(x + 2)$$
$$(2)(2x + 4)$$
$$(1)(4x + 8)$$

Factor # 1 Factor #2

Dec 7-6:20 PM

Factoring using the GCF

Factoring requires that we find the greatest common factor (GCF) for the terms in the product.

E.g. Factor the following:

$(8x + 12) = (\ \ )(\ \ \ + \ \ )$

Step #1: find the GCF for the numerical part of both terms 8x and 12

The GCF is 4

Step #2: Write "4" as your first factor

$(8x + 12) = (4)(\ \ \ + \ \ )$

Step #3: Mentally divide the **4** into each term. The answers make up the second factor (which go in the brackets).

result goes here

$\left(\frac{8x}{\mathbf{4}} + \frac{12}{\mathbf{4}}\right) = (\mathbf{4})(2x + 3)$

result goes here

Some factoring requires that we find a GCF for the variables as well.

E.g.#1 Factor the following:

$4x^2 + 6x$

Step #1: find the GCF for the "numerical" part of both terms $4x^2$ and 6x

Both terms have a factor of 2 so the GCF is **2**

Step #2: find the GCF for the 'variable' part of both terms $4x^2$ and 6x

Both terms have at least one x in each so the GCF is **x**

Step #3: The total GCF for $4x^2 + 6x$ is **2x**

Step #4: Mentally divide the **2x** into each term. The answers make up the second factor.

result goes here

$\left(\frac{4x^2}{\mathbf{2x}} + \frac{6x}{\mathbf{2x}}\right) = (\mathbf{2x})(2x + 3)$

result goes here

Dec 7-6:20 PM

Practice:

Principles & Process Text Pg.88 #5,6

Dec 7-6:39 PM

Please get a textbook from the back and open it to Pg.67.

Take out your homework questions. (Pg.67#13, Pg.88#5)

Dec 7-6:43 PM

Multiplying Binomials

Dec 3-10:22 PM

Multiplying Binomials

- To multiply two Binomials - each term in **each** binomial needs to be multiplied by **each other**.
- FOIL helps you keep track of multiplying Binomials.
- You need to remember your exponent laws for multiplying. (you add the exponent if they have the same **base**)

Dec 3-10:24 PM

FOIL

- F - first terms of each binomial
- O - Outside terms of each binomial
- I - Inside terms of each binomial
- L - Last terms of each binomial

Dec 3-10:24 PM

Example of FOIL

$(x - 2)(x + 3)$ These are the First Terms (F)

$(x - 2)(x + 3)$ These are the Outside Terms (O)

$(x - 2)(x + 3)$ These are the Inside Terms (I)

$(x - 2)(x + 3)$ These are the Last Terms (L)

Dec 3-10:24 PM

Example

$(g + 1)(g + 3)$

$g^2 + 3g + 1g + 3$

$\boxed{g^2 + 4g + 3}$

$(b - 2)(b - 3)$

$b^2 - 3b - 2b + 6$

$\boxed{b^2 - 5b + 6}$

$(y - 3)(2y + 1)$

$y^2 + y - 6y - 3$

$\boxed{y^2 - 5y - 3}$

Dec 3-10:26 PM

You Try

$(x + 4)(x - 5)$

$(x - 6)(x + 8)$

Dec 3-10:31 PM

You Try - SOLUTIONS

$(x + 4)(x - 5)$

$x^2 - 5x + 4x - 20$

$\boxed{x^2 - x - 20}$

$(x - 6)(x + 8)$

$x^2 + 8x - 6x - 48$

$\boxed{x^2 + 2x - 48}$

Dec 3-10:31 PM

Practice:

Principles & Process
Pg. 80 #1a-f, 2a-f

Dec 7-6:47 PM

Factoring Trinomials

Factoring trinomials requires that we understand the four steps of expansion.

To factor polynomials we must find:

- two numbers that multiply to give the product of the 3rd term and yet add up to give the sum of the 2nd term.

Dec 7-6:47 PM

5) Factoring $x^2 + bx + c$

STEPS	EXAMPLE $x^2 + 6x + 8$ (1) $x^2 + 5x - 24$ (2)
A) Write down (x)(x).	A) (x)(x)
B) Determine the signs to go in the brackets: $x^2 \pm bx + c$ *Sum* means *same signs*, If this is + then (x +)(x +) If this is − then (x −)(x −) $x^2 \pm bx - c$ *Difference* means *different signs*, The result is (x +)(x −) or (x −)(x +)	B_1) $x^2 + 6x + 8$ same signs sum (x +)(x +) B_2) $x^2 + 5x - 24$ difference (x +)(x −)
C) Determine the factors of the last term.	C_1) 8 $x^2 + 6x + 8$ 1 | 8 2 | 4 C_2) 24 $x^2 + 5x - 24$ 1 | 24 2 | 12 3 | 8 4 | 6
D) Determine the factors which either have a **sum** or a **difference** of the middle coefficient: $x^2 \pm bx \pm c$ If this is a **sum** (+) then look for a sum of the factors. If this is a **difference** (−) then look for a difference of the factors.	D_1) 8 $x^2 + 6x + 8$ 1 | 8 2 | 4 Sum of 6 D_2) 24 $x^2 + 5x - 24$ 1 | 24 2 | 12 3 | 8 4 | 6 Difference of 5
E) Place the factors in the empty positions of the two binomials. $x^2 \pm bx \pm c$ If this is a **sum** (+) then place factors in any order. If this is a **difference** (−) then place larger factor with the first sign of the trinomial.	E_1) $x^2 + 6x + 8$ (x + **2**)(x + **4**) or (x + **4**)(x + **2**) E_2) $x^2 + 5x - 24$ difference 8 > 3 therefore 8 goes with + sign (x + **8**)(x − **3**)

Dec 7-7:01 PM

Factor the following **completely**;

1) $c^2 + 25c + 24$

2) $y^2 + 7y + 10$

3) $w^2 - 8w + 7$

4) $g^2 - 12g + 35$

5) $p^2 + p - 12$

6) $a^2 + 6a - 27$

7) $a^2 - 5a - 14$

8) $m^2 - 8m - 65$

9) $3z^2 + 8z - 11$

10) $5a^2 + 12a + 4$

Dec 7-7:04 PM

Practice:

Principles & Practice
Pg. 93 #6abd,7ab,8abc

Dec 7-7:07 PM

How to factor a trinomial when the coefficient is more than one:

Factoring $ax^2 + bx + c$ (a > 1)

STEPS	EXAMPLE $3x^2 + 4x - 4$
A) Multiply the first and last terms.	A) $3x^2 + 4x - 4$ $(3)(-4) = \mathbf{-12}$
B) Determine the factors of the product from A) which add to get the middle term of the trinomial. Attach the variable to these two factors.	B) -12 $6x \quad -2x$
C) Write the first and last terms and place the answers from B) in the middle.	C) $3x^2 + 6x - 2x - 4$
D) Factor the first two and the last two terms.	D) $3x(x + 2) - 2(x + 2)$
E) Factor out a common **BINOMIAL** factor.	E) $(x + 2)(3x - 2)$

Dec 7-7:09 PM

9) $3z^2 + 8z - 11$

10) $5a^2 + 12a + 4$

11) $3w^2 - 10w + 8$

12) $4c^2 + 9c - 9$

13) $2t^2 + 4t - 48$

14) $3p^2 - 18p + 24$

15) $6a^2 - 8a - 14$

16) $20y^2 - 16y - 4$

Dec 7-7:18 PM

Practice:

Principles & Practice
Pg. 93 #6eg, 7cfg

Dec 7-7:07 PM

Attachments

Factoring Polynomials - notes.doc

Math 9 Factoring Assignment.doc

Factoring Trinomials worksheet.doc

Factoring GCF.doc